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THE
ASSURANCE MAGAZINE.

On a New Method of Constructing a Table of the Probabilities of Survivorship between Two Lives for every Combination of Ages, and also a Table of the Present Value of Survivorship Assurances of £1 on (x) against (y). By DAVID CHISHOLM, A.I.A., Actuary to the North British Assurance Company.

THE present paper was undertaken on happening to peruse very lately Messrs. Gray, Smith, and Orchard's laborious and useful work, entitled *Assurance and Annuity Tables, according to the Carlisle rate of Mortality, at 3 per Cent.* In the introductory section of that work, when referring to the larger Tables V. and VI., containing the single and annual premiums for survivorship assurances, it is there very justly remarked, "Considering the frequency with which occasion arises for the functions here tabulated, and the tedious nature of the operations requisite when their values have to be formed from other tables, it may, at first sight, seem surprising that complete tables of them had not been formed long ere now; but the wonder will cease when it is farther considered that, until recently, there had been pointed out no other method of forming them than the tedious operation just referred to. It had not been shown that one value might be made to help in the formation of the next, and that a complete table might in consequence be constructed at the cost of a much smaller amount of labour than would be requisite if each value had to be separately formed." And in a Postscript to the same work, the construction of similar tables, at Carlisle 4, 5, and 6 per cent. interest, on the same principle, or on another not yet promulgated, is there suggested.

It is hardly possible to overrate the importance of such tables as are here referred to, and all who have felt the want of them, and, since the publication of the above work, experienced the benefit

to be derived from consulting them, cannot fail to acquiesce in the surprise here expressed, that they had not been formed long ere now. It will doubtless be generally admitted, that the lack of complete tables of the kind has been more or less an inconvenience to the actuary, and is one which is daily becoming more serious by the rapid extension of life assurance throughout the kingdom: which is felt by the multiplied variety of questions that are in consequence presented to him for solution, and still further augmented by the increasing number of survivorship policies entered on the books of Insurance Companies for valuation at a periodical balance. A judicious selection of survivorship tables would at once remove this difficulty, and, in most instances, afford also the ready means of obtaining a correct solution of *three life* assurance questions, which, as found by the present mode, are at best only approximations. The object of this paper is therefore to render the construction of the tables suggested more practicable, if possible, than it is even by the method adopted in *Assurance and Annuity Tables*, by explaining to the best of my knowledge the method pursued by Mr. Edward Sang, late Actuary and Civil Engineer, Edinburgh, in constructing tables of this kind—a method which differs materially from all others known, both in the formulæ which form the basis of them, and in the mode of construction.

The Tables given as a specimen have been computed by this method, and were undertaken expressly for this paper. They are similar in principle to the Commutation Tables of Mr. Griffith Davies, and consequently possess, *mutatis mutandis*, the same properties; but the mode of forming them, besides being fundamentally logarithmic in its character, possesses this additional peculiarity, which is its distinguishing feature, that in adding two numbers together, the addend is not placed *under* the number to be increased, but to the right or left of that number, and the sum to the right of both, forming one horizontal line. So likewise in subtraction, the subtrahend is placed at the left hand, the minuend in the middle, and the remainder at the right hand. The arrangement for effecting this will be afterwards explained.

Before considering, however, the construction of the *Assurance Tables*, it will be more in order, first, to investigate the necessary formulæ, with reference to Survivorship Tables of Probability, under one problem; and, in a second problem, the other class of tables will fall to be considered as a very natural corollary to the first.

PROBLEM I.

To construct a Table of the Probabilities of Survivorship between two lives, (x) and (y), for every combination of ages.

Agreeably to the usual notation, l_x and l_y will denote the tabular numbers alive at the respective ages of (x) and (y). Let $l_x - l_{x+1} = d_x$, $l_{x+1} - l_{x+2} = d_{x+1}$, and so on, and similarly with (y); and let the problem be to determine the probability that, of these two lives, (x) shall be the first that fails.

Now, the number of deaths which happen out of those aged x during the first year, combined with the number of survivors of those aged y at the end of the year, is equal to the number of combinations *at the end of the year* in which those aged x may have died first. But, in the first year, there are d_x deaths of x , and there are l_{y+1} alive of y at the end of the year; hence the number of combinations is $d_x \times l_{y+1}$, which, in the case of an assurance on the life of (x), will express the number of payments we have to make for that year, (y) being then alive to draw them. But, again, in the course of the year there are $l_y - l_{y+1}$ deaths of class y , and (on the assumption that the deaths are uniformly distributed throughout the year in both classes, whatever be the difference of age) the chance is fair that one-half of them predecease the individuals of d_x ; hence, the total number of deaths expected of x first for that year, is $d_x \times \frac{1}{2}(l_y + l_{y+1}) = d_x \times l_{y+\frac{1}{2}}$.

By similar reasoning, it might be shown that the total number of deaths of x first, in the second year, is $d_{x+1} \times l_{y+\frac{3}{2}}$, and that in the n th year the number is $d_{x+n-1} \times l_{y+n-\frac{1}{2}}$. Hence, the sum of all these products will be the total number of cases in which the death of (x) shall happen first when combined with (y). And as $l_{x,y}$ is the number of combinations at the commencement, we obtain, as a final result for the probability that (x) shall die before (y), the expression

$$\frac{d_x \times l_{y+\frac{1}{2}} + d_{x+1} \times l_{y+\frac{3}{2}} + \dots + d_{x+n-1} \times l_{y+n-\frac{1}{2}}}{l_{x,y}} = S_{x,y}^1,$$

a very distinct symbol obtained from Gray's *Tables and Formulæ*.

* If we multiply the numerator and denominator of the above expression by the same quantity, or by $l_{x+1} \cdot l_{y+1}$, its value will not be altered, though its form will be changed; thus,

$$\frac{l_{x+1} \cdot l_{y+1}}{l_{x+1} \cdot l_{y+1}} \times \left(\frac{d_x \times l_{y+\frac{1}{2}} + d_{x+1} \times l_{y+\frac{3}{2}} + \&c.}{l_{x,y}} \right) = \frac{l_{x+1} \cdot l_{y+1}}{l_{x,y}} \times \left(\frac{d_x \times l_{y+\frac{1}{2}}}{l_{x+1} \cdot l_{y+1}} + S_{x+1,y+1}^1 \right) \\ = P_{x,y} \left[\frac{1}{2}(P_x^{-1} - 1) \cdot (P_y^{-1} + 1) + S_{x+1,y+1}^1 \right];$$

which is the expression given in *Tables and Formulæ* for the same function, $S_{x,y}^1$, found above.

On examining carefully the numerator of this expression, it will be observed to consist of a series of terms, which will be exhausted when the oldest life reaches the limit of the Mortality Table; and that the factors composing those terms are, on the one hand, d_x , and its successive values, and, on the other hand, $l_{y+\frac{1}{2}}$, and its successive values, both of which might be very conveniently arranged under one another in vertical columns for multiplication, and the products placed opposite to them. This is therefore the arrangement adopted; but as the multiplication is effected by means of logarithms, the logarithms of those functions were first obtained, and, being initial values, are set down in the Elementary Tables, Series I., entitled $\log. d_x$, and $\log. l_{x+\frac{1}{2}}$.

Their multiplication (or rather the addition of their logs.) was effected by the following arrangement:—A large sheet of drawing paper, of about 30 inches in length, was ruled across, the lines being equidistant, and rather less than a quarter of an inch asunder: every fifth line was ruled a degree brighter than the rest, and every tenth a degree brighter than the fifth, the fiftieth and hundredth lines being still brighter, care being taken to leave sufficient blank space at the top for writing thereon the symbol expressing the nature of the table. The annexed Tables are written on paper ruled in this manner; and although the ages are there set down for the sake of convenient reference, yet this need not be done in practice, as the mode of ruling is designed to serve for the correct ascertainment of the ages, which a little experience will enable any one to do very speedily and accurately, while the absence of them, in the construction of tables, to which this plan is chiefly applicable, is felt to be a great convenience.

The paper being thus ruled, was next cut into slips of about $1\frac{3}{8}$ of an inch wide, when it was ready to be used. In constructing the tables in Series III., the first thing required to be done was to write on one of those slips the logarithms of d_x , and on another the logs. of $l_{x+\frac{1}{2}}$, in the manner shown in Series I.; and as these must be added together, they are placed (vertically) on a table, side by side, and in contact with one another. As the difference between (x) and (y) is five years, (x) being the younger, $\log. l_{x+\frac{1}{2}}$ is moved five spaces above $\log. d_x$, since $\log. l_{y+\frac{1}{2}} = \log. l_{x+5+\frac{1}{2}}$. A blank slip is then placed at the right of these two, the lines of which are made to coincide with $\log. d_x$, and a weight is now placed on the top to keep them from moving as the work proceeds. The two sets of logs. were now added together, beginning at the top, until the whole of the blank slip was filled. This being done by two persons, the results (which have been named $\log. \gamma_{x-y}^1$) were

then compared ; and next, the natural numbers* taken out to seven places, taking care to arrange them in vertical columns of three figures deep, counting from the decimal point, in order that they might be the more easily added together or subtracted from one another. After the two sets of numbers were carefully compared, and the errors, if any, corrected, the next step was to sum them up, to accomplish which a blank slip was placed to the right of the slip titled $\gamma_{x,y}^1$, coinciding at top and bottom, and the result at the bottom ($\gamma_{99.104}^1$) transferred to the opposite space on the blank slip, after which the latter was lifted up one space, and a weight placed on the paper ; the two horizontal lines at the bottom were then added together, and the sum placed above the last result on the blank slip. This sum was then added to the number on the other slip, and the sum of these placed above the previous sum, and so on from the bottom to the top, until the whole slip was filled up. This function, which is also new, has been denoted by $\mu_{x,y}^1$. Of course, it will be necessary to check the work as it proceeds by a direct summation of all the values of $\gamma_{x,y}^1$ gone over. This should be done at least every ten lines ; and if these sums are reserved, the successive additions to them of each ten lines will check every tenth value of $\mu_{x,y}^1$, and all below it. The logarithms of $\mu_{x,y}^1$ were next obtained, being, of course, the logarithms of all the values of the numerator in the function

$$\frac{d_x \times l_{y+\frac{1}{2}} + d_{x+1} \times l_{y+1\frac{1}{2}} + \&c.}{l_{x,y}}.$$

The denominator $l_{x,y}$ (in logs.) was formed by writing log. l_x on two slips, and adding them together, one of them being lifted five spaces higher than the other to suit the difference in the ages, five years. The two slips, log. $l_{x,y}$ and log. $\mu_{x,y}^1$, were then placed together, coinciding at top and bottom, log. $l_{x,y}$ being at the left hand, and log. $\mu_{x,y}^1 - \log. l_{x,y}$ gave log. $S_{x,y}^1$, the numbers corresponding to which showed the probability of (x) dying before (y), thus completing Series III. of the Tables.

If we wish to find the probability of the opposite event happening, the function

$$\frac{d_y \times l_{x+\frac{1}{2}} + d_{y+1} \times l_{x+1\frac{1}{2}} + d_{y+2} \times l_{x+2\frac{1}{2}} + \&c.}{l_{x,y}}$$

* As the factors d_x and $l_{x+\frac{1}{2}}$ consist only of from two to five digits, their product, $\gamma_{x,y}^1$, might perhaps be as expeditiously obtained without the intervention of logarithms, by placing those two slips together according to the difference of age, with a blank slip at the right, and a weight on the top. The factors being then opposite one another may be multiplied together on scroll paper, and the results set down on the blank slip as they are found. This table, when completed, could then be used in connection with Table XL. of Mr. Jones' work.

may be shown by similar reasoning to express such probability, the symbol for which is $S_{x,y}^{-1}$. But if the other probability has been carefully ascertained, this function need not also be computed, the one being just the complement of the other. It has been very judiciously remarked, however, by Mr. Gray, in his *Tables and Formulæ*, that "in forming a table, it may be desirable to compute both probabilities independently of each other, to serve for mutual verification."

PROBLEM II.

To construct a Table of the Present Value of a Survivorship Assurance of £1 on the life of (x) against (y), payable at the end of the year in which the life of (x) fails, provided he shall have been survived by (y).

From the demonstration and explanations given in the last problem, it is not necessary in proceeding with this to do more than consider how the conclusions there arrived at will be affected by the interest of money; or, in other words, to show how the element v may be best combined with the expression

$$\frac{d_x \times l_{y+\frac{1}{2}} + d_{x+1} \times l_{y+1\frac{1}{2}} + d_{x+2} \times l_{y+2\frac{1}{2}} + \&c.}{l_{x,y}},$$

which is evidently all that is required in order to obtain the assurance $A_{x,y}^{-1}$.

On examining carefully the numerator of the above fraction with reference to an assurance on the life of (x), payable under the conditions named, it is plain that the first term, $d_x \times l_{y+\frac{1}{2}}$, will express the number of payments to be made for the first year, not, however, depending upon (y) surviving the *end* of the year, but on the simple condition of his having survived (x); and, in like manner, $d_{x+1} \times l_{y+1\frac{1}{2}}$, will express the number of payments to be made in the second year, and so on for the third and following years, the right to the benefit secured being determined in the middle of each year. But the supposition we make here, in conformity with the usual practice, is, that the payment is not due till the end of the year in which the life of (x) fails. Hence, if these several number of payments expected in each year be multiplied by the present value of £1 due at the end of the respective years, the expression for the value of an assurance of £1 on (x) against (y) will be

$$\frac{v d_x \times l_{y+\frac{1}{2}} + v^2 d_{x+1} \times l_{y+1\frac{1}{2}} + v^3 d_{x+2} \times l_{y+2\frac{1}{2}} + \&c.}{l_{x,y}},$$

wherein the several terms of the numerator express the present value of all the payments to be made in each year until the oldest life becomes extinct; while $l_{x,y}$ is the number of combinations alive of those aged x and y at the commencement.

The above fraction may be put into a shape more suitable for our purpose by multiplying numerator and denominator by v^x (which will not affect its value). The formula then becomes

$$\frac{v^{x+1}d_x \times l_{y+\frac{1}{2}} + v^{x+2}d_{x+1} \times l_{y+1\frac{1}{2}} + v^{x+3}d_{x+2} \times l_{y+2\frac{1}{2}} + \&c.}{v^x l_{x,y}};$$

or, in a more compact form,

$$\frac{* C_x \times l_{y+\frac{1}{2}} + C_{x+1} \times l_{y+1\frac{1}{2}} + C_{x+2} \times l_{y+2\frac{1}{2}} + \&c.}{D_{x,y}} = \Lambda_{x,y}^1.$$

This is therefore the working formula, the elementary tables required being C_x and $l_{y+\frac{1}{2}}$ in logarithms. Log. v^x was formed expressly for the annexed set of Tables before I had observed it in Table VIII. of *Assurance and Annuity Tables*; and the successive values were found by writing $\log v$ (9.987 1627 753) on a piece of card, and also at the top of one of the slips, cut a little broader than usual, and then adding it to itself all the way down. The three extra figures were added in order to obtain a correct figure for the seventh place, to which number of places it was ultimately reduced, and the whole was found to agree precisely with the value given in the Tables quoted. Log. d_x was then added to $\log v^x$ thus formed, producing $\log C_x$. In adding $\log C_x$ to $\log l_{y+\frac{1}{2}}$, it was only necessary to shift the elementary table, $\log l_{x+\frac{1}{2}}$ five spaces higher than $\log C_x$ to suit the difference of five years, the sum being, as formerly stated, placed on a slip at the right of these two; this new function has been called $\log C_{x,y}^1$, to which the

* If the numerator and denominator of this fraction be multiplied by $D_{x+1,y+1}$ its form will be changed into

$$\frac{D_{x+1,y+1}}{D_{x,y}} \times \left(\frac{C_x \times l_{y+\frac{1}{2}}}{D_{x+1,y+1}} + \Lambda_{x+1,y+1}^1 \right);$$

therefore,

$$\Lambda_{x,y}^1 = v p_{x,y} \times \left[\frac{1}{2} (p_x^{-1} - 1) \cdot (p_y^{-1} + 1) + \Lambda_{x+1,y+1}^1 \right],$$

which is Mr. Gray's formula, as given in his *Tables and Formulæ*.

This form of expression, from its compactness, possesses more of the character of an algebraical formula than the one found above, and has been accurately and extensively exemplified by the publication of *Assurance and Annuity Tables*, the offspring of this new and ingenious method of construction; but it must be borne in mind that Mr. Sang's object was to adapt his formula to the *commutation* form, which had already worked so successfully in the computation of his Single Life Tables. It is very probable, however, from his entire acquaintance with the subject, that he was quite aware of the connection thus shown to exist between any given value and the next younger, though such a formula would evidently not suit his object.

natural numbers were obtained, taking care to arrange them in columns of three figures deep right and left of the decimal point. The sum was next got in the way already described, and was denoted by $M_{x,y}^1$, to which the logarithms were then sought out. Lastly, $\log. M_{x,y}^1 - \log. D_{x,y}$ produced $\log. A_{x,y}^1$, the natural numbers to which gave the values required of an assurance of £1 on (x) against (y), as set down in Series IV. of the Tables.

In this manner, and by means of the elementary tables $\log. C_x$ and $\log. l_{x+y}$, the function $M_{x,y}^1$ may be computed for every difference of x and y , by moving $\log. l_{x+y}$ as many steps up as there is difference of age. But if the surviving life is the younger instead of the elder, the element v^x must be combined with l_{x+y} , producing D_{x+y} . The elementary tables for (y) against (x) are therefore $\log. D_{x+y}$ and $\log. d_x$; and in adding these together, it is only necessary to shift $\log. d_x$ the requisite number of spaces above $\log. D_{x+y}$, according to the difference between x and y , and proceed as in the last case. In Series V. of the Tables, this operation has also been performed for the difference, five years, so that Series IV. and V. are complementary values, whose sum is the common joint assurance. To each of these the functions $R_{x,y}^1$ and $R_{x,y}^{-1}$ have been added respectively, whereby an increasing assurance on (x) against (y) and on (y) against (x) may be found; and it is believed that this is the first time that these have been tabulated for contingent survivorship benefits.

The foregoing remarks, though, it may be, somewhat informally put together, will not, I think, fail to make it appear that the method just described for computing Survivorship Tables is preferable to that which formed the basis of Table V. in *Assurance and Annuity Tables*, and therefore the one that should be adopted in the event of an attempt being made to complete the two-life canon. By this system we not only arrive at the final result with, perhaps, less waste of figures than by the other mode, but we have in the subsidiary steps to it the additional functions $C_{x,y}^1$ and $M_{x,y}^1$, so useful in obtaining the value of temporary and deferred benefits, &c. We have also through them supplied the means of ascertaining with more accuracy and expedition than have hitherto been attainable, the single and annual premium for a reversionary annuity during the remainder of the life of B after the death of A, on condition that all the premiums shall be returned should B die before A.

In concluding this paper, I may take the liberty to mention that I was personally engaged, with many others, in the construction of Mr. Sang's Single Life Tables, published in 1841; and

that these Tables were computed by the columnar method, and in the manner just described.

It is well known to many that that work, since its publication, has attracted a considerable share of well-merited praise from many competent judges; but I think not a little is also due to its talented author for the ingenious method devised by him of using detached slips, sufficient in length to contain the whole Carlisle Table. By this method Mr. Sang was enabled to entrust, under his superintendence, a great part of the labour to persons of no higher arithmetical skill than such as decimal arithmetic and a practical knowledge of logarithms afforded, while great facility was also given for comparing each stage of the work before another step in advance was taken, the correctness of the last comparison thereby operating as a complete check on the whole.

By the same means, Mr. Sang had also advanced a considerable way in the construction of a series of Two-life Tables, with a view to publication as a second volume, showing by inspection the single and annual premium for most of the annuity and assurance benefits usually required in practice. The Tables of Single and Annual Premiums for $A_{x,y}^1$ and $A_{x,y}^{-1}$, were entirely completed, for every difference of age, by a formula similar to that deduced from Problem II. of this paper, the only difference being that the money was assumed to be payable at the *instant of death*,* in place of six

* The present value of a benefit payable *at death* is evidently greater than a similar benefit payable *six months after death*. And as the payments in the former case are, on an average, made six months earlier in each year, the expression for $A_{x,y}^1$ payable at death, which we may denote by $\mathcal{A}_{x,y}^1$, will become

$$\frac{v^{x+\frac{1}{2}}d_x \times l_{y+\frac{1}{2}} + v^{x+1\frac{1}{2}}d_{x+1} \times l_{y+1\frac{1}{2}} + v^{x+2\frac{1}{2}}d_{x+2} \times l_{y+2\frac{1}{2}} + \&c.}{v^x l_{x,y}},$$

and this is the formula from which Mr. Sang's tables were computed. But if this fraction be multiplied by $\sqrt{v}$, it will become

$$\frac{v^{x+1}d_x \times l_{y+\frac{1}{2}} + v^{x+2}d_{x+1} \times l_{y+1\frac{1}{2}} + \&c.}{v^x l_{x,y}} = A_{x,y}^1;$$

that is, $\mathcal{A}_{x,y}^1 \times \sqrt{v}$, or, which is the same thing, $\frac{\mathcal{A}_{x,y}^1}{\sqrt{(1+r)}} = A_{x,y}^1$, the assurance payable six months after death.

The same relation might be shown to subsist between Mr. Sang's single life assurance values, and the value of the same benefit due six months later. And although it is unfortunate that he made such an assumption when computing his Single Life Tables, there is not on that account such inconvenience in using them as might be supposed,

where a desire is felt to obtain the smaller value. For since $\log \sqrt{1.03} = \frac{0.0128372}{2} = 0.0064186$, the assurance values there tabulated may be easily converted into similar values payable six months later, by subtracting this quantity, or to suit his five place logs. 0.00642 from each column of logarithms, to which the corresponding natural num-

months after death. As far as I know, therefore, Mr. Sang has the merit of being the first to construct survivorship assurance tables by Barretts' method; and his computed tables would have been published at the time, had not circumstances occurred which led him to abandon his intention.

In the explanation given of the construction of the annexed Series of Tables by means of detached slips of pasteboard, it is not intended that it should be inferred that they can only be suc-

	$\Delta \log. p_x$		$\log. v p_x$
0	0.000 0000		
	0.036 0804	0	9.914 5845
	0.007 3474		9.950 6649
	0.012 3511		9.958 0123
	0.004 1428		9.970 3634
5	0.004 8556		9.974 5062
	0.002 4336	5	9.979 3618
	0.001 5305		9.981 7954
	0.000 9703		9.983 3259
	0.000 6537		9.984 2962
10	0.000 2589		9.984 9499
	9.999 8555	10	9.985 2088
	9.999 9216		9.985 0643
	9.999 9204		9.984 9859
	9.999 8504		9.984 9063
15	9.999 7093		9.984 7567
	9.999 7737	15	9.984 4660
	9.999 9098		9.984
	9.999 9790		
	9.999 9786		
20	9.999 9784		
	0.000 0503		
	9.999 9788		
	9.999 9785		
	9.999 9781		
25	* * * *		
	* * * *		

cessfully applied in the columnar mode of construction. On the contrary, it will at once appear that the system is of general application. Take, for instance, the example in page 43 of *Tables and Formulæ*, where the author, by his new and admirable method of constructing elementary tables by differences, describes the method of forming $\log. v p_x$. The annexed diagram shows how the same table may be formed by means of the slips. The table is here formed by beginning with the youngest; hence the difference column, $\Delta \log. p_x$, is the same as $\Delta \text{colog. } p_x$ in *Tables and Formulæ*, extended merely to seven places of figures. The initial value, 9.9145845 = $\log. v p_0$, is placed on the space for age 0, in the table $\log. v p_x$ to be formed, which is moved down one step opposite the first difference, and a weight put on at the top to keep them from moving. These two numbers are now added together, and the sum placed immediately below the initial value of the table form-

ing, which sum is added to the second difference, and this sum placed under the last as before, and so on until the whole table, $\log. v p_x$, is completed. The successive values of $\log. v p_x$ may be verified occasionally in the way pointed out in *Tables and Formulæ*, which, of course, will check the whole of the previous work.

The function $\log. p_{(x,y)}$ may be formed by adding together two columns of $\log. p_x$ written on slips, moving one of them upwards,

bers will be the values required: or the value for a single case may be found very nearly by dividing by 1.0149, a number easily remembered.

according to the difference of x and y , and placing the sum on a third slip. If one of the two former be $\log. vp_x$, the third slip will then contain the successive values of $\log. vp_{(x,y)}$; and it is obvious that by this method of formation, the different values of $\log. p_{(x,y)}$ and $\log. vp_{(x,y)}$, where x and y have the same difference, will be found under one another in the same vertical column, in place of on several horizontal lines of different columns. A considerable saving of time and figures is also effected by this mode.

A similar method may be adopted for the formation of the different values of $\log. \frac{1}{2}(p_x^{-1}-1)(p_y^{-1}+1)$. From what has been said under Problem I., this function is equal to $\log. d_x + \log. l_{y+1} - \log. l_{x+1} + \log. l_{y+1}$, that is, equal to $\log. \gamma_{x,y}^1 - \log. l_{x+1.y+1}$; and as the terms in which $y-x$ is constant, are, by our mode, always ranged under one another on the same slip, all therefore that requires to be attended to before subtracting is to move the slip, containing the different values of $\log. l_{x,y}$, *one step upward*, and place the difference on a third slip at the right of these, which difference will be the successive values of $\log. \frac{1}{2}(p_x^{-1}-1)(p_y^{-1}+1)$. Care must obviously be taken that the slips used are the right ones, viz., those constructed for corresponding differences of x and y .

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POSTSCRIPT.

Since the foregoing was written, I find that the annexed Series of Tables have been computed without adverting to the circumstance of Jones' Joint-Life Commutation Tables having been constructed on the principle of discounting money to the birth of the *eldest*. There does not appear to be any ground stated in Jones' work for this preference, though it would seem more natural to discount to the birth of the *youngest*, since the combination of two given lives cannot be traced farther back than that date. But whatever mode be adopted, the ratio of the D, N, and S columns not being destroyed, it follows that either class of tables will give the same result in finding the value of annuities.

Mr. Sang's practice was invariably to discount to the *birth of the youngest*; and accordingly Table A, computed on this principle, has been added to the annexed Series, to facilitate the application of the formulæ contained in this paper. The column $N_{x,y}$, as was usual with him, is tabulated one step lower than in Jones' tables,

being in the form in which $\frac{N_{x,y}}{D_{x,y}}$ produces $1 + a_{x,y}$, that is, the joint annuity payable per advance. But though the functions $M_{x,y}^1$ and $M_{x,y}^{-1}$ can be more conveniently used in connection with Table A, they may, by multiplying by v^5 , be used also with Jones' Table XXVII. with the same accuracy. Their relation may be seen thus:—

Single Premium, (x) against (y).		Annual Premium, (x) against (y).	
Jones' Table.	Table A.	Jones' Table.	Table A.
$\frac{v^5 M_{x,y}^1}{D_{x,y}}$	$\frac{M_{x,y}^1}{D_{x,y}}$	$\frac{v^5 M_{x,y}^1}{N_{x-1,y-1}}$	$\frac{M_{x,y}^1}{N_{x,y}}$

My object having been in this paper to suggest the construction of tables of the form $C_{x,y}^1$ and $M_{x,y}^1$, with their complementary values, $C_{x,y}^{-1}$ and $M_{x,y}^{-1}$, as supplementary to Jones' voluminous series, it is quite clear, therefore, that if any such are ever constructed, the discount must be to the *birth of the eldest* in place of the youngest. Log. $C_{x,y}^1$ will hence be formed from the elementary tables log. d_x and log. $D_{x+\frac{1}{2}}$, the latter being moved upwards, according to the difference of age, while the former is stationary. Log. $C_{x,y}^{-1}$, on the other hand, will be formed from the tables log. $l_{x+\frac{1}{2}}$ and log. C_x , the latter being moved upwards, and the former continuing stationary. The functions $C_{x,y}^1$, $M_{x,y}^1$, &c., will of course be obtained as before, and may then be used in connection with Jones' tables without any inconvenience.

The following formulæ will illustrate the application of the tables, being a recapitulation of the results obtained in Problem II., with some additions easily deduced therefrom.

$$\frac{M_{x,y}^1}{D_{x,y}} = A_{x,y}^1 = \text{assurance on (x) against (y).}$$

$$\frac{M_{x,y}^1}{N_{x,y}} = \frac{A_{x,y}^1}{1 + a_{x,y}} = \text{annual premium for the same.}$$

$$\frac{M_{x,y}^{-1}}{D_{x,y}} = A_{x,y}^{-1} = \text{assurance on (y) against (x).}$$

$$\frac{M_{x,y}^{-1}}{N_{x,y}} = \frac{A_{x,y}^{-1}}{1 + a_{x,y}} = \text{annual premium for the same.}$$

$$\frac{R_{x,y}^1}{D_{x,y}} = \text{increasing assurance on (x) against (y).}$$

$$\frac{R_{x,y}^1}{N_{x,y}} = \text{annual premium for the same.}$$

$$\frac{M_{x,y}^1}{D_{x,y} - M_{x,y}^1} = \text{assurance on } (x) \text{ against } (y), \text{ with return of premium should } (y) \text{ predecease } (x).$$

$$\frac{M_{x,y}^1}{N_{x,y} - R_{x,y}^1} = \text{annual premium for the same.}$$

$$\frac{M_{x,y}^1 - M_{x+n,y+n}^1}{D_{x,y}} = \text{assurance on } (x) \text{ against } (y) \text{ for } n \text{ years.}$$

$$\frac{M_{x,y}^1 - M_{x+n,y+n}^1}{N_{x,y} - N_{x+n,y+n}} = \text{annual premium for the same.}$$

$$a_x - a_{x,y} = \frac{N_{x+1}}{D_x} - \frac{N_{x+1,y+1}}{D_{x,y}} = \frac{l_y N_{x+1} - N_{x+1,y+1}}{D_{x,y}} = \text{value of a reversionary annuity on } (x) \text{ after the death of } (y). \text{ Hence,}$$

$$\frac{l_y N_{x+1} - N_{x+1,y+1}}{D_{x,y} - M_{x,y}^1} = \text{value of the same annuity, with return of premium should } (x) \text{ predecease } (y).$$

$$\frac{l_y N_{x+1} - N_{x+1,y+1}}{N_{x,y} - R_{x,y}^1} = \text{annual premium for the same.}$$

$$a_y - a_{x,y} = \frac{N_{y+1}}{D_y} - \frac{N_{x+1,y+1}}{D_{x,y}} = \frac{l_x N_{y+1} - N_{x+1,y+1}}{v^5 D_{x,y}} = \text{value of a reversionary annuity on the life } (y) \text{ after the death of } (x). \text{ Hence,}$$

$$\frac{l_x N_{y+1} - N_{x+1,y+1}}{v^5 D_{x,y} - M_{x,y}^1} = \text{value of the same annuity, with return of premium, should } (y) \text{ predecease } (x).$$

$$\frac{l_x N_{y+1} - N_{x+1,y+1}}{v^5 N_{x,y} - R_{x,y}^1} = \text{annual premium for the same.}$$

D. C.

The several columns comprising Series III., IV., and V. of the Tables which follow, are set down in the same order in which they are produced when computing, *and also in the same form*, that a more perfect knowledge of the whole process may be obtained. It would thus appear, from the function $M_{x,y}^1$ being the third in order, that only three columns need be computed for each difference of age. And as these columns become shorter as the difference increases, fifty-five slips for each function will be found sufficient to contain the whole Carlisle Table: hence, if we include tables of the opposite event, $\frac{1}{x,y}$, the whole number of slips required for each rate of interest will be $55 \times 3 \times 2 = 330$ ($\frac{2}{3}$ only of which would require to be tabulated), each seldom containing more than 99 values. If the functions $R_{x,y}^1$ and $R_{x,y}^1$ be added, the number of slips would of course be 110 more.

SERIES I.—Elementary Tables.

Age of	Log. $1.03^{-x} =$	Log. $(l_x - l_{x+1}) =$	Log. $\frac{l_x + l_{x+1}}{2} =$	Log. $d_x +$ Log. $v^{x+1} =$	Log. $l_{x+\frac{1}{2}} +$ Log. $v^{x+\frac{1}{2}} =$
(x)	Log. v^x	Log. d_x	Log. $l_{x+\frac{1}{2}}$	Log. C_x	Log. $D_{x+\frac{1}{2}}$
0	0.000 0000	3.187 2386	3.965 2252	3.174 4014	3.952 3880
1	9.987 1628	2.833 7844	3.909 5560	2.808 1100	3.883 8816
2	9.974 3256	2.703 2914	3.876 5931	2.664 7797	3.838 0814
3	9.961 4883	2.440 9091	3.853 4548	2.389 5602	3.802 1059
4	9.948 6511	2.303 1961	3.838 6917	2.239 0100	3.774 5056
5	9.935 8139	2.082 7854	3.828 4343	2.005 7621	3.751 4110
6	9.922 9767	1.913 8139	3.821 8409	1.823 9533	3.731 9803
7	9.910 1394	1.763 4280	3.817 2347	1.660 7302	3.714 5369
8	9.897 3022	1.633 4685	3.813 8811	1.517 9335	3.698 3461
9	9.884 4650	1.518 5139	3.811 3404	1.390 1417	3.682 9682
10	9.871 6278	1.462 3980	3.809 2566	1.321 1885	3.668 0471
11	9.858 7905	1.491 3617	3.807 2305	1.337 3150	3.653 1838
12	9.845 9533	1.505 1500	3.805 0929	1.338 2661	3.638 2090
13	9.833 1161	1.518 5139	3.802 8763	1.338 7928	3.623 1552
14	9.820 2789	1.544 0680	3.800 5453	1.351 5096	3.607 9869
15	9.807 4416	1.591 0646	3.797 9942	1.385 6690	3.592 5986
16	9.794 6044	1.623 2493	3.795 1846	1.405 0165	3.576 9518
17	9.781 7672	1.633 4685	3.792 2165	1.402 3985	3.561 1465
18	9.768 9300	1.633 4685	3.789 1928	1.389 5612	3.545 2855
19	9.756 0927	1.633 4685	3.786 1478	1.376 7240	3.529 4033
20	9.743 2555	1.633 4685	3.783 0814	1.363 8868	3.513 4997
21	9.730 4183	1.623 2493	3.780 0291	1.340 8304	3.497 6102
22	9.717 5811	1.623 2493	3.776 9916	1.327 9931	3.481 7354
23	9.704 7438	1.623 2493	3.773 9326	1.315 1559	3.465 8392
24	9.691 9066	1.623 2493	3.770 8520	1.302 3187	3.449 9214
25	9.679 0694	1.633 4685	3.767 7123	1.299 7007	3.433 9445
26	9.666 2322	1.633 4685	3.764 5124	1.286 8634	3.417 9073
27	9.653 3949	1.653 2125	3.761 2134	1.293 7702	3.401 7711
28	9.640 5577	1.698 9700	3.757 6237	1.326 6905	3.385 3442
29	9.627 7205	1.748 1880	3.753 5831	1.363 0713	3.368 4664
30	9.614 8833	1.755 8749	3.749 2337	1.357 9209	3.351 2797
31	9.602 0460	1.755 8749	3.744 8013	1.345 0837	3.334 0101
32	9.589 2088	1.748 1880	3.740 3627	1.324 5596	3.316 7343
33	9.576 3716	1.740 3627	3.735 9580	1.303 8971	3.299 4924
34	9.563 5344	1.740 3627	3.731 5485	1.291 0598	3.282 2456
35	9.550 6971	1.740 3627	3.727 0937	1.278 2226	3.264 9536
36	9.537 8599	1.748 1880	3.722 5517	1.273 2107	3.247 5744
37	9.525 0227	1.755 8749	3.717 8784	1.268 0604	3.230 0639
38	9.512 1855	1.763 4280	3.713 0703	1.262 7762	3.212 4185
39	9.499 3482	1.785 3298	3.708 0383	1.271 8408	3.194 5493
40	9.486 5110	1.819 5439	3.702 6028	1.293 2177	3.176 2766
41	9.473 6738	1.838 8491	3.696 7494	1.299 6857	3.157 5860
42	9.460 8366	1.851 2583	3.690 5947	1.299 2576	3.138 5940
43	9.447 9993	1.851 2583	3.684 2617	1.286 4204	3.119 4238
44	9.435 1621	1.851 2583	3.677 8350	1.273 5832	3.100 1599
45	9.422 3249	1.845 0980	3.671 3580	1.254 5857	3.080 8457
46	9.409 4877	1.838 8491	3.664 8769	1.235 4995	3.061 5273
47	9.396 6504	1.826 0748	3.658 4407	1.209 8880	3.042 2539
48	9.383 8132	1.799 3405	3.652 1980	1.170 3165	3.023 1740
49	9.370 9760	1.785 3298	3.646 1586	1.143 4686	3.004 2974
50	9.358 1388	1.770 8520	3.640 2329	1.116 1535	2.985 5344
51	9.345 3015	1.792 3917	3.634 1749	1.124 8560	2.966 6392
52	9.332 4643	1.812 9134	3.627 7242	1.132 5405	2.947 3513

SERIES I.—Elementary Tables.

Age of	Log. $1.03^{-x} =$	Log. $(l_x - l_{x+1}) =$	Log. $\frac{l_x + l_{x+1}}{2} =$	Log. $d_x +$ Log. $v^{x+1} =$	Log. $l_{x+1} +$ Log. $v^{x+1} =$
(x)	Log. v^x	Log. d_x	Log. $l_{x+\frac{1}{2}}$	Log. C_x	Log. $D_{x+\frac{1}{2}}$
53	9.319 6271	1.832 5089	3.620 8645	1.139 2988	2.927 6544
54	9.306 7899	1.845 0980	3.613 6304	1.139 0506	2.907 5830
55	9.293 9526	1.863 3229	3.606 0050	1.144 4383	2.887 1204
56	9.281 1154	1.880 8136	3.597 9145	1.149 0918	2.866 1927
57	9.268 2782	1.913 8139	3.589 1674	1.169 2549	2.844 6084
58	9.255 4410	1.968 4829	3.579 2690	1.211 0866	2.821 8727
59	9.242 6037	2.025 3059	3.567 7320	1.255 0724	2.797 4985
60	9.229 7665	2.086 3598	3.554 1256	1.303 2891	2.771 0549
61	9.216 9293	2.100 3705	3.538 8250	1.304 4626	2.742 9171
62	9.204 0921	2.103 8037	3.522 6398	1.295 0585	2.713 8946
63	9.191 2548	2.096 9100	3.505 8958	1.275 3276	2.684 3134
64	9.178 4176	2.096 9100	3.488 6212	1.262 4904	2.654 2016
65	9.165 5804	2.093 4217	3.470 7044	1.246 1649	2.623 4476
66	9.152 7432	2.089 9051	3.452 1699	1.229 8110	2.592 0758
67	9.139 9059	2.089 9051	3.432 8892	1.216 9738	2.559 9579
68	9.127 0687	2.089 9051	3.412 7125	1.204 1366	2.526 9440
69	9.114 2315	2.092 4217	3.391 4644	1.194 8160	2.492 8587
70	9.101 3943	2.093 4217	3.369 0302	1.181 9787	2.457 5872
71	9.088 5570	2.127 1048	3.344 3923	1.202 8246	2.420 1121
72	9.075 7198	2.164 3529	3.315 9703	1.227 2355	2.378 8529
73	9.062 8826	2.193 1246	3.283 0750	1.243 1700	2.333 1204
74	9.050 0454	2.220 1081	3.245 0189	1.257 3162	2.282 2270
75	9.037 2081	2.204 1200	3.202 7607	1.228 4909	2.227 1316
76	9.024 3709	2.193 1246	3.157 4568	1.204 6583	2.168 9905
77	9.011 5337	2.164 3529	3.109 2410	1.163 0494	2.107 9375
78	8.998 6965	2.120 5739	3.059 5634	1.106 4331	2.045 4226
79	8.985 8592	2.107 2100	3.007 3210	1.080 2320	1.980 3430
80	8.973 0220	2.064 4580	2.951 8230	1.024 6428	1.912 0078
81	8.960 1848	2.049 2180	2.892 6510	0.996 5656	1.839 9986
82	8.947 3476	2.008 6002	2.828 6599	0.943 1105	1.763 1702
83	8.934 5103	1.973 1279	2.760 4225	0.894 8010	1.682 0956
84	8.921 6731	1.924 2793	2.687 5290	0.833 1152	1.596 3649
85	8.908 8359	1.892 0946	2.608 5260	0.788 0933	1.504 5247
86	8.895 9987	1.851 2583	2.520 4835	0.734 4198	1.403 6450
87	8.883 1615	1.806 1800	2.421 6039	0.676 5042	1.291 9281
88	8.870 3242	1.707 5702	2.314 9201	0.565 0572	1.172 4071
89	8.857 4870	1.591 0646	2.208 1725	0.435 7144	1.052 8223
90	8.844 6498	1.568 2017	2.091 6670	0.400 0143	0.923 4796
91	8.831 8126	1.477 1213	1.954 2425	0.296 0966	0.773 2178
92	8.818 9753	1.322 2193	1.809 5597	0.128 3574	0.615 6978
93	8.806 1381	1.146 1280	1.672 0979	9.939 4289	0.465 3988
94	8.793 3009	1.000 0000	1.544 0680	9.780 4637	0.324 5317
95	8.780 4637	0.845 0980	1.423 2459	9.612 7244	0.190 8723
96	8.767 6264	0.698 9700	1.311 7539	9.453 7592	0.066 5431
97	8.754 7892	0.602 0600	1.204 1200	9.344 0120	9.946 0720
98	8.741 9520	0.477 1213	1.096 9100	9.206 2361	9.826 0248
99	8.729 1148	0.301 0300	1.000 0000	9.017 3075	9.716 2775
100	8.716 2775	0.301 0300	0.903 0900	9.004 4703	9.606 5303
101	8.703 4403	0.301 0300	0.778 1513	8.991 6331	9.468 7544
102	8.690 6031	0.301 0300	0.602 0600	8.978 7959	9.279 8259
103	8.677 7659	0.301 0300	0.301 0300	8.965 9586	8.965 9586
104	8.664 9286	0.000 0000	9.698 9700	8.652 0914	8.351 0614

SERIES II.—Elementary Tables.

Age of		Log. l_x + Log. $l_y =$	Log. $l_{x,y}$ + Log. $v^x =$	Age of		Log. l_x + Log. $l_y =$	Log. $l_{x,y}$ + Log. $v^x =$
(x)	(y)	Log. $l_{x,y}$	Log. $D_{x,y}$	(x)	(y)	Log. $l_{x,y}$	Log. $D_{x,y}$
0	5	7·832 3173	7·832 3173	50	55	7·253 0709	6·611 2097
1	6	7·751 9380	7·739 1008	51	56	7·239 3495	6·584 6510
2	7	7·710 0727	7·684 3983	52	57	7·224 7667	6·557 2310
3	8	7·677 0853	7·638 5736	53	58	7·208 9426	6·528 5697
4	9	7·657 4193	7·606 0704	54	59	7·191 2303	6·498 0202
5	10	7·642 5498	7·578 3637	55	60	7·171 3736	6·465 3262
6	11	7·632 7948	7·555 7715	56	61	7·148 7260	6·429 8414
7	12	7·625 3289	7·535 4683	57	62	7·124 5688	6·392 8470
8	13	7·619 3151	7·516 6173	58	63	7·098 8394	6·354 2804
9	14	7·614 1920	7·498 6570	59	64	7·071 2598	6·313 8635
10	15	7·609 5730	7·481 2008	60	65	7·041 1784	6·270 9449
11	16	7·604 9222	7·463 7127	61	66	7·008 1645	6·225 0938
12	17	7·599 9006	7·445 8539	62	67	6·973 4763	6·177 5684
13	18	7·594 7104	7·427 8265	63	68	6·937 2000	6·128 4548
14	19	7·589 4196	7·409 6985	64	69	6·899 6058	6·078 0234
15	20	7·583 9578	7·391 3994	65	70	6·860 1114	6·025 6918
16	21	7·578 1837	7·372 7881	66	71	6·818 8615	5·971 6047
17	22	7·572 2336	7·354 0008	67	72	6·773 6587	5·913 5646
18	23	7·566 1721	7·335 1021	68	73	6·723 2961	5·850 3648
19	24	7·560 0681	7·316 1608	69	74	6·667 3152	5·781 5467
20	25	7·553 9208	7·297 1763	70	75	6·604 4070	5·705 8013
21	26	7·547 6553	7·278 0736	71	76	6·537 7756	5·626 3326
22	27	7·541 4165	7·258 9976	72	77	6·464 2417	5·539 9615
23	28	7·534 9816	7·239 7254	73	78	6·384 2389	5·447 1215
24	29	7·528 1175	7·220 0241	74	79	6·298 8795	5·348 9249
25	30	7·520 7366	7·199 8060	75	80	6·203 1077	5·240 3158
26	31	7·513 1385	7·179 3707	76	81	6·103 1381	5·127 5090
27	32	7·505 4715	7·158 8664	77	82	5·993 5575	5·005 0912
28	33	7·497 6629	7·138 2206	78	83	5·878 3488	4·877 0453
29	34	7·489 4812	7·117 2017	79	84	5·757 2814	4·743 1406
30	35	7·480 7599	7·095 6432	80	85	5·627 4529	4·600 4749
31	36	7·471 8723	7·073 9183	81	86	5·487 3916	4·447 5764
32	37	7·462 8100	7·052 0188	82	87	5·331 6297	4·278 9773
33	38	7·453 6480	7·030 0196	83	88	5·159 9760	4·094 4863
34	39	7·444 3838	7·007 9182	84	89	4·981 1343	3·902 8074
35	40	7·434 7628	6·985 4599	85	90	4·800 6483	3·709 4842
36	41	7·424 6001	6·962 4600	86	91	4·585 8554	3·481 8541
37	42	7·413 9689	6·938 9916	87	92	4·346 3530	3·229 5145
38	43	7·402 9417	6·915 1272	88	93	4·097 8818	2·968 2060
39	44	7·391 6852	6·891 0334	89	94	3·859 7386	2·717 2256
40	45	7·380 0216	6·866 5326	90	95	3·629 4096	2·474 0594
41	46	7·367 8572	6·841 5310	91	96	3·382 9171	2·214 7297
42	47	7·355 3503	6·816 1869	92	97	3·130 3338	1·949 3091
43	48	7·342 6743	6·790 6736	93	98	2·878 5218	1·684 6599
44	49	7·330 2003	6·765 3624	94	99	2·643 4527	1·436 7536
45	50	7·317 7421	6·740 0670	95	100	2·431 3638	1·211 8275
46	51	7·305 3957	6·714 8834	96	101	2·206 8258	0·974 4522
47	52	7·292 6611	6·689 3115	97	102	1·954 2425	0·709 0317
48	53	7·279 6197	6·663 4329	98	103	1·623 2493	0·365 2013
49	54	7·266 4550	6·637 4310	99	104	1·041 3927	9·770 5075

SERIES III.—*Probability of (x) before (y).*

Age of		Log. d_x + Log. $l_{y+\frac{1}{2}}$	Natural Numbers to Log. $\gamma_{x,y}^1$	Age of		Log. d_x + Log. $l_{y+\frac{1}{2}}$	Natural Numbers to Log. $\gamma_{x,y}^1$
(x)	(y)	Log. $\gamma_{x,y}^1$	$\gamma_{x,y}^1$	(x)	(y)	Log. $\gamma_{x,y}^1$	$\gamma_{x,y}^1$
0	5	7.015 6729	10 367 473	50	55	5.376 8570	238 153.5
1	6	6.655 6253	4 525 070	51	56	5.390 3062	245 644.0
2	7	6.520 5261	3 315 325	52	57	5.402 0808	252 395.0
3	8	6.254 7902	1 798 002	53	58	5.411 7779	258 094.0
4	9	6.114 5365	1 301 777	54	59	5.412 8300	258 720.0
5	10	5.892 0420	779 905.5	55	60	5.417 4485	261 486.0
6	11	5.721 0444	526 071.0	56	61	5.419 6386	262 808.0
7	12	5.568 5209	370 272.0	57	62	5.436 4537	273 183.0
8	13	5.436 3448	273 114.5	58	63	5.474 3787	298 111.5
9	14	5.319 0592	208 477.5	59	64	5.513 9271	326 533.0
10	15	5.260 3922	182 134.5	60	65	5.557 0642	360 632.0
11	16	5.286 5463	193 440.0	61	66	5.552 5404	356 895.0
12	17	5.297 3665	198 320.0	62	67	5.536 6929	344 106.5
13	18	5.307 7067	203 098.5	63	68	5.509 6225	323 312.5
14	19	5.330 2158	213 902.5	64	69	5.488 3744	307 875.0
15	20	5.374 1460	236 671.5	65	70	5.462 4519	290 036.0
16	21	5.403 2784	253 092.0	66	71	5.434 2974	271 830.0
17	22	5.410 4601	257 312.0	67	72	5.405 8754	254 610.0
18	23	5.407 4011	255 506.0	68	73	5.372 9801	236 037.0
19	24	5.404 3205	253 700.0	69	74	5.338 4406	217 992.0
20	25	5.401 1808	251 872.5	70	75	5.296 1824	197 780.0
21	26	5.387 7617	244 209.0	71	76	5.284 5616	192 558.0
22	27	5.384 4627	242 361.0	72	77	5.273 5939	187 756.0
23	28	5.380 8730	240 366.0	73	78	5.252 6880	178 932.0
24	29	5.376 8324	238 140.0	74	79	5.227 4291	168 822.0
25	30	5.382 7022	241 380.5	75	80	5.155 9430	143 200.0
26	31	5.378 2698	238 929.5	76	81	5.085 7756	121 836.0
27	32	5.393 5752	247 500.0	77	82	4.993 0128	98 404.0
28	33	5.434 9280	272 225.0	78	83	4.880 9964	76 032.0
29	34	5.479 7365	301 812.0	79	84	4.794 7390	62 336.0
30	35	5.482 9686	304 066.5	80	85	4.672 9840	47 096.0
31	36	5.478 4266	300 903.0	81	86	4.569 7015	37 128.0
32	37	5.466 0664	292 460.0	82	87	4.430 2041	26 928.0
33	38	5.453 4330	284 075.0	83	88	4.288 0480	19 411.0
34	39	5.448 4010	280 802.5	84	89	4.132 4518	13 566.0
35	40	5.442 9655	277 310.0	85	90	3.983 7616	9 633.0
36	41	5.444 9374	278 572.0	86	91	3.805 5008	6 390.0
37	42	5.446 4696	279 556.5	87	92	3.615 7397	4 128.0
38	43	5.447 6897	280 343.0	88	93	3.379 6681	2 397.0
39	44	5.463 1648	290 512.5	89	94	3.135 1326	1 365.0
40	45	5.490 9019	309 672.0	90	95	2.991 4476	980.5
41	46	5.503 7260	318 952.5	91	96	2.788 8752	615.0
42	47	5.509 6990	323 369.5	92	97	2.526 3393	336.0
43	48	5.503 4563	318 754.5	93	98	2.243 0380	175.0
44	49	5.497 4169	314 352.5	94	99	2.000 0000	100.0
45	50	5.485 3309	305 725.0	95	100	1.748 1880	56.0
46	51	5.473 0240	297 183.0	96	101	1.477 1213	30.0
47	52	5.453 7990	284 314.5	97	102	1.204 1200	16.0
48	53	5.420 2050	263 151.0	98	103	0.778 1513	6.0
49	54	5.398 9602	250 588.0	99	104	0.000 0000	1.0

SERIES III.—Probability of (x) before (y) .

Age of		Sum of $\gamma_{x,y}^1$ from x, y to the limit of the Table =		Age of		Sum of $\gamma_{x,y}^1$ from x, y to the limit of the Table =	
(x)	(y)	$\mu_{x,y}^1$	Log. $\mu_{x,y}^1$	(x)	(y)	$\mu_{x,y}^1$	Log. $\mu_{x,y}^1$
0	5	41 322 591·	7·616 1876	50	55	72 364 67·5	6·859 5267
1	6	30 955 118·	7·490 7324	51	56	69 983 14·0	6·844 9935
2	7	26 430 048·	7·422 0980	52	57	67 526 70·0	6·829 4756
3	8	23 114 723·	7·363 8887	53	58	65 002 75·0	6·812 9317
4	9	21 316 721·	7·323 7204	54	59	62 421 81·0	6·795 3364
5	10	20 014 944·	7·301 3544	55	60	59 834 61·0	6·776 9525
6	11	19 235 038·5	7·284 0931	56	61	57 219 75·0	6·757 5460
7	12	18 708 967·5	7·272 0499	57	62	54 591 67·0	6·737 1264
8	13	18 338 695·5	7·263 3685	58	63	51 859 84·0	6·714 8311
9	14	18 065 581·0	7·256 8519	59	64	48 878 72·5	6·689 1198
10	15	17 857 103·5	7·251 8110	60	65	45 613 39·5	6·659 0924
11	16	17 674 969·0	7·247 3587	61	66	42 007 07·5	6·623 3225
12	17	17 481 529·0	7·242 5794	62	67	38 438 12·5	6·584 7622
13	18	17 283 209·0	7·237 6243	63	68	34 997 06·0	6·544 0315
14	19	17 080 110·5	7·232 4907	64	69	31 763 93·5	6·501 9343
15	20	16 866 208·0	7·227 0174	65	70	28 685 18·5	6·457 6577
16	21	16 629 536·5	7·220 8801	66	71	25 784 82·5	6·411 3641
17	22	16 376 444·5	7·214 2196	67	72	23 066 52·5	6·362 9822
18	23	16 119 132·5	7·207 3417	68	73	20 520 42·5	6·312 1863
19	24	15 863 626·5	7·200 4024	69	74	18 160 05·5	6·259 1171
20	25	15 609 926·5	7·193 4008	70	75	15 980 13·5	6·203 5805
21	26	15 358 054·0	7·186 3362	71	76	14 002 33·5	6·146 2005
22	27	15 113 845·0	7·179 3750	72	77	12 076 75·5	6·081 9503
23	28	14 871 484·0	7·172 3543	73	78	10 199 19·5	6·008 5659
24	29	14 631 118·0	7·165 2775	74	79	8 409 87·5	5·924 7896
25	30	14 392 978·0	7·158 1507	75	80	6 721 65·5	5·827 4762
26	31	14 151 597·5	7·150 8055	76	81	5 289 65·5	5·723 4273
27	32	13 912 668·0	7·143 4104	77	82	4 071 29·5	5·609 7325
28	33	13 665 168·0	7·135 6150	78	83	3 087 25·5	5·489 5725
29	34	13 392 943·0	7·126 8761	79	84	2 326 93·5	5·366 7842
30	35	13 091 131·0	7·116 9771	80	85	1 703 57·5	5·231 3613
31	36	12 787 064·5	7·106 7709	81	86	1 232 61·5	5·090 8275
32	37	12 486 161·5	7·096 4289	82	87	861 33·5	4·935 1721
33	38	12 193 701·5	7·086 1356	83	88	592 05·5	4·772 3621
34	39	11 909 626·5	7·075 8982	84	89	397 94·5	4·599 8231
35	40	11 628 824·0	7·065 5358	85	90	262 28·5	4·418 7735
36	41	11 351 514·0	7·055 0538	86	91	165 95·5	4·219 9904
37	42	10 072 942·0	7·044 2630	87	92	102 05·5	4·008 8343
38	43	10 793 385·5	7·033 1577	88	93	60 77·5	3·783 7250
39	44	10 513 042·5	7·021 7284	89	94	36 80·5	3·565 9068
40	45	10 222 530·0	7·009 5584	90	95	23 15·5	3·364 6448
41	46	9 912 858·0	6·996 1989	91	96	13 35·0	3·125 4813
42	47	9 593 905·5	6·981 9955	92	97	7 20·0	2·857 3325
43	48	9 270 536·0	6·967 1049	93	98	3 84·0	2·584 3312
44	49	8 951 781·5	6·951 9095	94	99	2 09·0	2·320 1463
45	50	8 637 429·0	6·936 3345	95	100	1 09·0	2·037 4265
46	51	8 331 704·0	6·920 7338	96	101	53·0	1·724 2759
47	52	8 034 521·0	6·904 9601	97	102	23·0	1·361 7278
48	53	7 750 206·5	6·889 3133	98	103	7·0	0·845 0980
49	54	7 487 055·5	6·874 3110	99	104	1·0	0·000 0000

SERIES III.—*Probability of (x) before (y).*

Age of		Log. $\mu_{x,y}^1$ —Log. $l_{x,y}$ —	Probability that (x) shall die before (y), denoted by	Age of		Log. $\mu_{x,y}^1$ —Log. $l_{x,y}$ —	Probability that (x) shall die before (y) denoted by
(x)	(y)	Log. $S_{x,y}^1$	$S_{x,y}^1$	(x)	(y)	Log. $S_{x,y}^1$	$S_{x,y}^1$
0	5	9.783 8703	.60 795 34	50	55	9.606 4558	.40 406 93
1	6	9.733 7944	.54 801 75	51	56	9.605 6440	.40 331 46
2	7	9.712 0253	.51 525 87	52	57	9.604 7089	.40 244 72
3	8	9.686 8034	.48 618 71	53	58	9.603 9891	.40 178 07
4	9	9.671 3011	.46 913 85	54	59	9.604 1061	.40 188 90
5	10	9.658 8046	.45 583 17	55	60	9.605 5789	.40 325 42
6	11	9.651 2983	.44 802 09	56	61	9.608 8200	.40 627 49
7	12	9.646 7210	.44 332 38	57	62	9.612 5576	.40 978 64
8	13	9.644 0534	.44 060 91	58	63	9.615 9917	.41 303 96
9	14	9.642 6599	.43 919 76	59	64	9.617 8600	.41 482 03
10	15	9.642 2380	.43 877 11	60	65	9.617 9140	.41 487 19
11	16	9.642 4365	.43 897 17	61	66	9.615 1580	.41 224 75
12	17	9.642 6788	.43 921 67	62	67	9.611 2859	.40 858 83
13	18	9.642 9139	.43 945 44	63	68	9.606 8315	.40 441 90
14	19	9.643 0711	.43 961 36	64	69	9.602 3285	.40 024 74
15	20	9.643 0596	.43 960 19	65	70	9.597 5463	.39 586 43
16	21	9.642 6964	.43 923 44	66	71	9.592 5026	.39 129 35
17	22	9.641 9860	.43 851 66	67	72	9.589 3235	.38 843 95
18	23	9.641 1696	.43 769 30	68	73	9.588 8902	.38 805 23
19	24	9.640 3343	.43 685 20	69	74	9.591 8019	.39 066 27
20	25	9.639 4800	.43 599 35	70	75	9.599 1735	.39 735 03
21	26	9.638 6809	.43 519 20	71	76	9.608 4249	.40 590 54
22	27	9.637 9585	.43 446 87	72	77	9.617 7086	.41 467 57
23	28	9.637 3727	.43 388 31	73	78	9.624 3270	.42 104 35
24	29	9.637 1600	.43 367 06	74	79	9.625 9101	.42 258 12
25	30	9.637 4141	.43 392 44	75	80	9.624 3685	.42 108 38
26	31	9.637 6670	.43 417 72	76	81	9.620 2892	.41 714 71
27	32	9.637 9389	.43 444 91	77	82	9.616 1750	.41 321 40
28	33	9.637 9521	.43 446 23	78	83	9.611 2237	.40 852 97
29	34	9.637 3949	.43 390 52	79	84	9.609 5028	.40 691 42
30	35	9.636 2172	.43 273 02	80	85	9.603 9084	.40 170 61
31	36	9.634 8986	.43 141 83	81	86	9.603 4359	.40 126 93
32	37	9.633 6189	.43 014 90	82	87	9.603 5424	.40 136 76
33	38	9.632 4876	.42 902 99	83	88	9.612 3861	.40 962 47
34	39	9.631 5144	.42 806 96	84	89	9.618 6888	.41 561 27
35	40	9.630 7730	.42 733 95	85	90	9.618 1252	.41 507 37
36	41	9.630 4537	.42 702 54	86	91	9.634 1350	.43 066 05
37	42	9.630 2941	.42 686 85	87	92	9.662 4813	.45 970 71
38	43	9.630 2160	.42 679 18	88	93	9.685 8432	.48 511 33
39	44	9.630 0432	.42 662 20	89	94	9.706 1682	.50 835 62
40	45	9.629 5368	.42 612 48	90	95	9.735 2352	.54 354 46
41	46	9.628 3417	.42 495 38	91	96	9.742 5642	.55 279 51
42	47	9.626 6452	.42 329 70	92	97	9.726 9985	.53 333 33
43	48	9.624 4306	.42 114 40	93	98	9.705 8094	.50 793 65
44	49	9.621 7092	.41 851 33	94	99	9.676 6936	.47 500 00
45	50	9.618 6424	.41 556 83	95	100	9.606 0627	.40 370 37
46	51	9.615 3381	.41 241 85	96	101	9.517 4501	.32 919 27
47	52	9.612 2990	.40 954 26	97	102	9.407 4853	.25 555 55
48	53	9.609 6936	.40 709 30	98	103	9.221 8487	.16 666 67
49	54	9.607 8560	.40 537 41	99	104	8.958 6073	.09 090 91

SERIES IV.—Assurance on (x) against (y) .

Age of		Log. $l_{y+\frac{1}{2}}$ + Log. $C_x =$	Natural Numbers.	Age of		Log. $l_{y+\frac{1}{2}}$ + Log. $C_x =$	Natural Numbers.
(x)	(y)	Log. $C_{x,y}^1$	$C_{x,y}^1$	(x)	(y)	Log. $C_{x,y}^1$	$C_{x,y}^1$
0	5	7.002 8357	10 065 508.	50	55	4.722 1585	52 742.24
1	6	6.629 9509	4 265 313.	51	56	4.722 7705	52 816.61
2	7	6.482 0144	3 033 991.	52	57	4.721 7079	52 687.54
3	8	6.203 4413	1 597 501.	53	58	4.718 5678	52 307.96
4	9	6.050 3504	1 122 924.	54	59	4.706 7826	50 907.60
5	10	5.815 0187	653 158.6	55	60	4.698 5639	49 953.27
6	11	5.631 1838	427 743.9	56	61	4.687 9168	48 743.50
7	12	5.465 8231	292 296.1	57	62	4.691 8947	49 192.02
8	13	5.320 8098	209 319.5	58	63	4.716 9824	52 117.36
9	14	5.190 6870	155 126.9	59	64	4.743 6936	55 423.45
10	15	5.119 1827	131 577.8	60	65	4.773 9935	59 428.33
11	16	5.132 4996	135 674.9	61	66	4.756 6325	57 099.53
12	17	5.130 4826	135 046.3	62	67	4.727 9477	53 450.00
13	18	5.127 9856	134 272.0	63	68	4.688 0401	48 757.34
14	19	5.137 6574	137 295.9	64	69	4.653 9548	45 076.98
15	20	5.168 7504	147 485.8	65	70	4.615 1951	41 228.27
16	21	5.185 0456	153 124.8	66	71	4.574 2033	37 514.85
17	22	5.179 3901	151 143.7	67	72	4.532 9441	34 114.90
18	23	5.163 4938	145 711.5	68	73	4.487 2116	30 705.18
19	24	5.147 5760	140 467.6	69	74	4.439 8349	27 531.82
20	25	5.131 5991	135 393.9	70	75	4.384 7394	24 251.54
21	26	5.105 3428	127 450.9	71	76	4.360 2814	22 923.52
22	27	5.089 2065	122 802.3	72	77	4.336 4765	21 700.84
23	28	5.072 7796	118 244.1	73	78	4.302 7334	20 078.60
24	29	5.055 9018	113 737.0	74	79	4.264 6372	18 392.35
25	30	5.048 9344	111 926.9	75	80	4.180 3139	15 146.56
26	31	5.031 6647	107 563.4	76	81	4.097 3093	12 511.50
27	32	5.034 1329	108 176.5	77	82	3.991 7093	9 810.91
28	33	5.062 6485	115 517.7	78	83	3.866 8556	7 359.623
29	34	5.094 6198	124 342.5	79	84	3.767 7610	5 858.157
30	35	5.085 0146	121 622.7	80	85	3.633 1688	4 297.034
31	36	5.067 6354	116 851.8	81	86	3.517 0491	3 288.888
32	37	5.042 4380	110 265.1	82	87	3.364 7144	2 315.871
33	38	5.016 9674	103 984.21	83	88	3.209 7211	1 620.769
34	39	4.999 0981	99 792.55	84	89	3.041 2877	1 099.734
35	40	4.980 8254	95 680.92	85	90	2.879 7603	758.159
36	41	4.969 9601	93 316.84	86	91	2.688 6623	488.272 6
37	42	4.958 6551	90 919.09	87	92	2.486 0639	306.241 4
38	43	4.947 0379	88 519.28	88	93	2.237 1551	172.645 4
39	44	4.949 6758	89 058.60	89	94	1.979 7824	95.451 42
40	45	4.964 5757	92 167.04	90	95	1.823 2602	66.567 18
41	46	4.964 5626	92 164.29	91	96	1.607 8505	40.536 90
42	47	4.957 6983	90 719.00	92	97	1.332 4774	21.501 92
43	48	4.938 6184	86 819.72	93	98	1.036 3389	10.872 74
44	49	4.919 7418	83 126.94	94	99	0.780 4637	6.032 033
45	50	4.894 8186	78 490.77	95	100	0.515 8144	3.279 551
46	51	4.869 6744	74 075.47	96	101	0.231 9105	1.705 731
47	52	4.837 6122	68 803.76	97	102	9.946 0720	.883 226 4
48	53	4.791 1810	61 827.40	98	103	9.507 2661	.321 563 0
49	54	4.757 0990	57 160.90	99	104	8.716 2775	.052 032 84

SERIES IV.—Assurance on (x) against (y) .

Age of		Sum of $C_{x,y}^1$ from x, y , to the end of the Table.	Sum of $M_{x,y}^1$ from x, y , to the end of the Table.	Age of		Sum of $C_{x,y}^1$	Sum of $M_{x,y}^1$
(x)	(y)	$M_{x,y}^1$	$R_{x,y}^1$	(x)	(y)	$M_{x,y}^1$	$R_{x,y}^1$
0	5	27 339 631·	245 264 204·	50	55	1 124 427·17	13 608 314·06
1	6	17 274 123·	217 924 573·	51	56	1 071 684·93	12 483 886·89
2	7	13 008 810·	200 650 450·	52	57	1 018 868·32	11 412 201·96
3	8	9 974 819·	187 641 640·	53	58	966 180·78	10 393 333·64
4	9	8 377 318·	177 666 821·	54	59	913 872·82	9 427 152·86
5	10	7 254 394·	169 289 503·0	55	60	862 965·22	8 513 280·04
6	11	6 601 235·4	162 035 109·0	56	61	813 011·95	7 650 314·82
7	12	6 173 491·5	155 433 873·6	57	62	764 268·45	6 837 302·87
8	13	5 881 195·4	149 260 382·1	58	63	715 076·43	6 073 034·42
9	14	5 671 875·9	143 379 186·7	59	64	662 959·07	5 357 957·99
10	15	5 516 749·0	137 707 310·8	60	65	607 535·62	4 694 998·92
11	16	5 385 171·2	132 190 561·8	61	66	548 107·29	4 087 463·30
12	17	5 249 496·3	126 805 390·6	62	67	491 007·76	3 539 356·01
13	18	5 114 450·0	121 555 894·3	63	68	437 557·76	3 048 348·25
14	19	4 980 178·0	116 441 444·3	64	69	388 800·42	2 610 790·49
15	20	4 842 882·1	111 461 266·3	65	70	343 723·44	2 221 990·07
16	21	4 695 396·3	106 618 384·2	66	71	302 495·17	1 878 266·63
17	22	4 542 271·5	101 922 987·9	67	72	264 980·32	1 575 771·46
18	23	4 391 127·8	97 380 716·4	68	73	230 865·42	1 310 791·14
19	24	4 245 416·3	92 989 588·6	69	74	200 160·24	1 079 925·72
20	25	4 104 948·7	88 744 172·3	70	75	172 628·42	879 765·48
21	26	3 969 554·8	84 639 223·6	71	76	148 376·88	707 137·06
22	27	3 842 103·9	80 669 668·8	72	77	125 453·36	558 760·18
23	28	3 719 301·6	76 827 564·9	73	78	103 752·52	433 306·82
24	29	3 601 057·5	73 108 263·3	74	79	83 673·92	329 554·30
25	30	3 487 320·5	69 507 205·8	75	80	65 281·57	245 880·38
26	31	3 375 393·6	66 019 885·3	76	81	50 135·01	180 598·81
27	32	3 267 830·2	62 644 491·7	77	82	37 623·51	130 463·80
28	33	3 159 653·7	59 376 661·5	78	83	27 812·60	92 840·29
29	34	3 044 136·0	56 217 007·8	79	84	20 452·976	65 027·69
30	35	2 919 793·5	53 172 871·8	80	85	14 594·819	44 574·711
31	36	2 798 170·8	50 253 078·3	81	86	10 297·785	29 979·892
32	37	2 681 319·0	47 454 907·5	82	87	7 008·897	19 682·107
33	38	2 571 053·95	44 773 588·53	83	88	4 693·026	12 673·210
34	39	2 467 069·74	42 202 534·58	84	89	3 072·257	7 980·184
35	40	2 367 277·19	39 735 464·84	85	90	1 972·523	4 907·927
36	41	2 271 596·27	37 368 187·65	86	91	1 214·363 7	2 935·404
37	42	2 178 279·43	35 096 591·38	87	92	726·091 1	1 721·040 6
38	43	2 087 360·34	32 918 311·95	88	93	419·849 7	994·949 5
39	44	1 998 841·06	30 830 951·61	89	94	247·204 3	575·099 8
40	45	1 909 782·46	28 832 110·55	90	95	151·752 88	327·895 50
41	46	1 817 615·42	26 922 328·09	91	96	85·185 70	176·142 62
42	47	1 725 451·13	25 104 712·67	92	97	44·648 80	90·956 92
43	48	1 634 732·13	23 379 261·54	93	98	23·146 88	46·308 12
44	49	1 547 912·41	21 744 529·41	94	99	12·274 137	23·161 245
45	50	1 464 785·47	20 196 617·00	95	100	6·242 104	10·887 108
46	51	1 386 294·70	18 731 831·53	96	101	2·962 553	4·645 004
47	52	1 312 219·23	17 345 536·83	97	102	1·256 822 2	1·662 450 8
48	53	1 243 415·47	16 033 317·60	98	103	·373 595 8	·425 628 6
49	54	1 181 588·07	14 789 902·13	99	104	·052 032 84	·052 032 84

SERIES IV.—Assurance on (x) against (y) .

Age of		Log. $M_{x,y}^1 -$ Log. $D_{x,y} =$		Age of		Log. $M_{x,y}^1 -$ Log. $D_{x,y} =$	
(x)	(y)	Log. $M_{x,y}^1$	Log. $A_{x,y}^1$	(x)	(y)	Log. $M_{x,y}^1$	Log. $A_{x,y}^1$
0	5	7.436 7926	9.604 4753	50	55	6.050 9313	9.439 7216
1	6	7.237 3960	9.498 2952	51	56	6.030 0671	9.445 4161
2	7	7.114 2375	9.429 8392	52	57	6.008 1180	9.450 8870
3	8	6.998 9050	9.360 3314	53	58	5.985 0584	9.456 4887
4	9	6.923 1050	9.317 0346	54	59	5.960 8858	9.462 8656
5	10	6.860 6011	9.282 2374	55	60	5.935 9933	9.470 6671
6	11	6.819 6253	9.263 8538	56	61	5.910 0969	9.480 2555
7	12	6.790 5309	9.255 0626	57	62	5.883 2460	9.490 3990
8	13	6.769 4656	9.252 8483	58	63	5.854 3525	9.500 0721
9	14	6.753 7268	9.255 0698	59	64	5.821 4867	9.507 6232
10	15	6.741 6832	9.260 4824	60	65	5.783 5717	9.512 6268
11	16	6.731 1995	9.267 4868	61	66	5.738 8655	9.513 7717
12	17	6.720 1176	9.274 2637	62	67	5.691 0884	9.513 5200
13	18	6.708 7990	9.280 9725	63	68	5.641 0353	9.512 5805
14	19	6.697 2449	9.287 5464	64	69	5.589 7268	9.511 7034
15	20	6.685 1039	9.293 7045	65	70	5.536 2091	9.510 5173
16	21	6.671 6723	9.298 8842	66	71	5.480 7184	9.509 1137
17	22	6.657 2731	9.303 2723	67	72	5.423 2136	9.509 6490
18	23	6.642 5761	9.307 4740	68	73	5.363 3589	9.512 9941
19	24	6.627 9202	9.311 7594	69	74	5.301 3778	9.519 8311
20	25	6.613 3077	9.316 1314	70	75	5.237 1124	9.531 3111
21	26	6.598 7418	9.320 6682	71	76	5.171 3663	9.545 0337
22	27	6.584 5691	9.325 5715	72	77	5.098 4823	9.558 5208
23	28	6.570 4614	9.330 7360	73	78	5.015 9987	9.568 8772
24	29	6.556 4301	9.336 4060	74	79	4.922 5901	9.573 6652
25	30	6.542 4919	9.342 6859	75	80	4.814 7907	9.574 4749
26	31	6.528 3244	9.348 9537	76	81	4.700 1411	9.572 6321
27	32	6.514 2595	9.355 3931	77	82	4.575 4593	9.570 3681
28	33	6.499 6395	9.361 4189	78	83	4.444 2416	9.567 1963
29	34	6.483 4641	9.366 2624	79	84	4.310 7565	9.567 6159
30	35	6.465 3521	9.369 7089	80	85	4.164 1987	9.563 7238
31	36	6.446 8742	9.372 9559	81	86	4.012 7438	9.565 1674
32	37	6.428 3485	9.376 3297	82	87	3.845 6497	9.566 6724
33	38	6.410 1113	9.380 0917	83	88	3.671 4530	9.576 9667
34	39	6.392 1814	9.384 2632	84	89	3.487 4576	9.584 6502
35	40	6.374 2491	9.388 7892	85	90	3.295 0221	9.585 5379
36	41	6.356 3312	9.393 8712	86	91	3.084 3488	9.602 4947
37	42	6.338 1136	9.399 1220	87	92	2.860 9912	9.631 4767
38	43	6.319 5981	9.404 4709	88	93	2.623 0939	9.654 8879
39	44	6.300 7782	9.409 7448	89	94	2.393 0560	9.675 8304
40	45	6.280 9839	9.414 4513	90	95	2.181 1369	9.707 0775
41	46	6.259 5020	9.417 9710	91	96	1.930 3667	9.715 6370
42	47	6.236 9027	9.420 7158	92	97	1.649 8098	9.700 5007
43	48	6.213 4467	9.422 7731	93	98	1.364 4924	9.679 8325
44	49	6.189 7464	9.424 3840	94	99	1.088 9909	9.652 2373
45	50	6.165 7740	9.425 7070	95	100	0.795 3310	9.583 5035
46	51	6.141 8556	9.426 9722	96	101	0.471 6661	9.497 2139
47	52	6.118 0064	9.428 6949	97	102	0.099 2739	9.390 2422
48	53	6.094 6163	9.431 1834	98	103	9.572 4020	9.207 2007
49	54	6.072 4661	9.435 0351	99	104	8.716 2775	8.945 7700

SERIES IV.—Assurance on (x) against (y) .

Age of		Present value of an Assurance of £1 on the Life of (x) against (y) .	Age of		Present value of an Assurance of £1 on the Life of (x) against (y) .
(x)	(y)	$A_{x,y}^1$	(x)	(y)	$A_{x,y}^1$
0	5	·40 223 07	50	55	·27 524 64
1	6	·31 498 89	51	56	·27 887 92
2	7	·26 905 39	52	57	·28 241 45
3	8	·22 926 16	53	58	·28 608 08
4	9	·20 750 79	54	59	·29 031 24
5	10	·19 153 03	55	60	·29 557 46
6	11	·18 359 20	56	61	·30 217 29
7	12	·17 991 30	57	62	·30 931 36
8	13	·17 899 80	58	63	·31 628 03
9	14	·17 991 60	59	64	·32 182 75
10	15	·18 217 23	60	65	·32 555 68
11	16	·18 513 43	61	66	·32 641 62
12	17	·18 804 58	62	67	·32 622 71
13	18	·19 097 33	63	68	·32 552 21
14	19	·19 388 60	64	69	·32 486 54
15	20	·19 665 48	65	70	·32 397 93
16	21	·19 901 43	66	71	·32 293 40
17	22	·20 103 53	67	72	·32 333 22
18	23	·20 298 97	68	73	·32 583 22
19	24	·20 500 26	69	74	·33 100 24
20	25	·20 707 68	70	75	·33 986 86
21	26	·20 925 13	71	76	·35 077 91
22	27	·21 162 72	72	77	·36 184 35
23	28	·21 415 88	73	78	·37 057 59
24	29	·21 697 32	74	79	·37 468 41
25	30	·22 013 34	75	80	·37 538 33
26	31	·22 333 34	76	81	·37 379 38
27	32	·22 666 95	77	82	·37 185 03
28	33	·22 983 65	78	83	·36 914 44
29	34	·23 241 41	79	84	·36 950 13
30	35	·23 426 58	80	85	·36 620 46
31	36	·23 602 39	81	86	·36 742 39
32	37	·23 786 45	82	87	·36 869 93
33	38	·23 993 39	83	88	·37 754 32
34	39	·24 224 97	84	89	·38 428 21
35	40	·24 478 75	85	90	·38 506 84
36	41	·24 766 87	86	91	·40 040 06
37	42	·25 068 13	87	92	·42 803 25
38	43	·25 378 79	88	93	·45 173 94
39	44	·25 688 86	89	94	·47 405 68
40	45	·25 968 77	90	95	·50 942 17
41	46	·26 180 08	91	96	·51 956 16
42	47	·26 346 07	92	97	·50 176 53
43	48	·26 471 17	93	98	·47 844 54
44	49	·26 569 54	94	99	·44 899 06
45	50	·26 650 60	95	100	·38 326 89
46	51	·26 728 35	96	101	·31 420 56
47	52	·26 834 59	97	102	·24 560 79
48	53	·26 988 79	98	103	·16 113 90
49	54	·27 229 21	99	104	·08 826 124

SERIES V.—Assurance on (y) against (x).

Age of		Log. $D_{x+\frac{1}{2}}$ + Log. $d_y =$		Age of		Log. $D_{x+\frac{1}{2}}$ + Log. $d_y =$	
(x)	(y)	Log. $C_{x,y}^{-1}$	$C_{x,y}^{-1}$	(x)	(y)	Log. $C_{x,y}^{-1}$	$C_{x,y}^{-1}$
0	5	6.035 1734	1 084 359.8	50	55	4.848 8573	70 608.55
1	6	5.797 6955	627 618.1	51	56	4.847 4528	70 380.57
2	7	5.601 5094	399 493.2	52	57	4.861 1652	72 638.22
3	8	5.435 5744	272 630.5	53	58	4.896 1373	78 729.47
4	9	5.293 0195	196 344.9	54	59	4.932 8889	85 681.86
5	10	5.213 8090	163 609.7	55	60	4.973 4802	94 076.30
6	11	5.223 3420	167 240.7	56	61	4.966 5632	92 589.80
7	12	5.219 6869	165 839.1	57	62	4.948 4121	88 799.82
8	13	5.216 8600	164 763.1	58	63	4.918 7827	82 943.56
9	14	5.227 0362	168 669.4	59	64	4.894 4085	78 416.69
10	15	5.259 1117	181 598.3	60	65	4.864 4766	73 194.19
11	16	5.276 4331	188 987.5	61	66	4.832 8222	68 049.08
12	17	5.271 6775	186 929.3	62	67	4.803 7997	63 650.19
13	18	5.256 6237	180 560.9	63	68	4.774 2185	59 459.13
14	19	5.241 4554	174 363.4	64	69	4.747 6233	55 927.23
15	20	5.226 0671	168 293.4	65	70	4.716 8693	52 103.78
16	21	5.200 2011	158 562.7	66	71	4.719 1806	52 381.82
17	22	5.184 3958	152 895.9	67	72	4.724 3108	53 004.27
18	23	5.168 5348	147 412.7	68	73	4.720 0686	52 489.04
19	24	5.152 6526	142 119.2	69	74	4.712 9668	51 637.69
20	25	5.146 9682	140 271.1	70	75	4.661 7072	45 888.85
21	26	5.131 0787	135 231.8	71	76	4.613 2367	41 042.78
22	27	5.134 9479	136 441.9	72	77	4.543 2058	34 930.58
23	28	5.164 8092	146 153.5	73	78	4.453 6943	28 424.59
24	29	5.198 1094	157 800.9	74	79	4.389 4370	24 515.29
25	30	5.189 8194	154 817.3	75	80	4.291 5896	19 569.95
26	31	5.173 7822	149 204.6	76	81	4.218 2085	16 527.55
27	32	5.149 9591	141 240.5	77	82	4.116 5377	13 077.89
28	33	5.125 7069	133 569.4	78	83	4.018 5505	10 436.394
29	34	5.108 8291	128 478.1	79	84	3.904 6223	8 028.276
30	35	5.091 6424	123 493.0	80	85	3.804 1024	6 369.457
31	36	5.082 1981	120 836.5	81	86	3.691 2569	4 911.983
32	37	5.072 6092	118 197.7	82	87	3.569 3502	3 709.798
33	38	5.062 9204	115 590.0	83	88	3.389 6658	2 452.821
34	39	5.067 5754	116 835.6	84	89	3.187 4295	1 539.677
35	40	5.084 4975	121 478.0	85	90	3.072 7264	1 182.297
36	41	5.086 4235	122 017.9	86	91	2.880 7663	759.917 2
37	42	5.081 3222	120 593.0	87	92	2.614 1474	411.289 3
38	43	5.063 6768	115 791.5	88	93	2.318 5351	208.226 1
39	44	5.045 8076	111 123.9	89	94	2.052 8223	112.933 4
40	45	5.021 3746	105 044.8	90	95	1.768 5776	58.691 83
41	46	4.996 4351	99 182.5	91	96	1.472 1878	29.661 13
42	47	4.964 6688	92 186.8	92	97	1.217 7578	16.510 41
43	48	4.918 7643	82 940.06	93	98	0.942 5201	8.760 322
44	49	4.885 4897	76 822.72	94	99	0.625 5617	4.222 422
45	50	4.851 6977	71 071.87	95	100	0.491 9023	3.103 861
46	51	4.853 9190	71 436.32	96	101	0.367 5731	2.331 165
47	52	4.855 1673	71 641.93	97	102	0.247 1020	1.766 452
48	53	4.855 6829	71 727.03	98	103	0.127 0548	1.339 846
49	54	4.849 3954	70 696.10	99	104	9.716 2775	0.520 323

SERIES V.—Assurance on (y) against (x).

Age of		Sum of $C_{x,y}^{-1}$	Sum of $M_{x,y}^{-1}$	Age of		Sum of $C_{x,y}^{-1}$	Sum of $M_{x,y}^{-1}$
(x)	(y)	$M_{x,y}^{-1}$	$R_{x,y}^{-1}$	(x)	(y)	$M_{x,y}^{-1}$	$R_{x,y}^{-1}$
0	5	10 175 196·8	255 886 442·9	50	55	1 660 988·72	20 003 520·77
1	6	9 090 837·0	245 711 246·1	51	56	1 590 380·17	18 342 542·05
2	7	8 463 218·9	236 620 409·1	52	57	1 519 999·60	16 852 160·88
3	8	8 063 725·7	228 157 190·2	53	58	1 447 361·38	15 232 162·28
4	9	7 791 095·2	220 093 464·5	54	59	1 368 631·91	13 784 800·90
5	10	7 594 750·3	212 302 369·3	55	60	1 282 950·05	12 416 168·99
6	11	7 431 140·6	204 707 619·0	56	61	1 188 873·75	11 133 218·94
7	12	7 263 899·9	197 276 478·4	57	62	1 096 283·95	9 944 345·19
8	13	7 098 060·8	190 012 578·5	58	63	1 007 484·13	8 848 061·24
9	14	6 933 297·7	182 914 517·7	59	64	924 540·57	7 840 577·11
10	15	6 764 628·3	175 981 220·0	60	65	846 123·88	6 916 036·54
11	16	6 583 030·0	169 216 591·7	61	66	772 929·69	6 069 912·66
12	17	6 394 042·5	162 633 561·7	62	67	704 880·61	5 296 982·97
13	18	6 207 113·2	156 239 519·2	63	68	641 230·42	4 592 102·36
14	19	6 026 552·3	150 032 406·0	64	69	581 771·29	3 950 871·94
15	20	5 852 188·9	144 005 853·7	65	70	525 844·06	3 369 100·65
16	21	5 683 895·5	138 153 664·8	66	71	473 740·28	2 843 256·59
17	22	5 525 332·8	132 469 769·3	67	72	421 358·46	2 369 516·31
18	23	5 372 436·9	126 944 436·5	68	73	368 354·19	1 948 157·85
19	24	5 225 024·2	121 571 999·6	69	74	315 865·15	1 579 803·66
20	25	5 082 905·0	116 346 975·4	70	75	264 227·46	1 263 938·51
21	26	4 942 633·9	111 264 070·4	71	76	218 338·61	999 711·05
22	27	4 807 402·1	106 321 436·5	72	77	177 295·83	781 372·44
23	28	4 670 960·2	101 514 034·4	73	78	142 365·25	604 076·61
24	29	4 524 806·7	96 843 074·2	74	79	113 940·66	461 711·36
25	30	4 367 005·8	92 318 267·5	75	80	89 425·37	347 770·70
26	31	4 212 188·5	87 951 261·7	76	81	69 855·42	258 345·33
27	32	4 062 983·9	83 739 073·2	77	82	53 327·87	188 489·91
28	33	3 921 743·4	79 676 089·3	78	83	40 249·98	135 162·044
29	34	3 788 174·0	75 754 345·9	79	84	29 813·583	94 912·067
30	35	3 659 695·9	71 966 171·9	80	85	21 785·307	65 098·484
31	36	3 536 202·9	68 306 475·0	81	86	15 415·850	43 313·177
32	37	3 415 366·4	64 770 273·1	82	87	10 503·867	27 897·327
33	38	3 297 168·7	61 354 906·7	83	88	6 794·069	17 393·460
34	39	3 181 578·7	58 057 737·0	84	89	4 341·248	10 599·391
35	40	3 064 743·1	54 876 159·3	85	90	2 801·571	6 258·143
36	41	2 943 265·1	51 811 416·2	86	91	1 619·274	3 456·572 4
37	42	2 821 247·2	48 868 151·1	87	92	859·356 57	1 837·298 6
38	43	2 700 654·2	46 046 903·9	88	93	448·067 27	977·942 0
39	44	2 584 862·7	43 346 249·7	89	94	239·841 17	529·874 73
40	45	2 473 738·8	40 761 387·0	90	95	126·907 77	290·033 56
41	46	2 368 694·0	38 287 648·2	91	96	68·215 94	163·125 79
42	47	2 269 511·5	35 918 954·2	92	97	38·554 81	94·909 85
43	48	2 177 324·75	33 649 442·73	93	98	22·044 396	56·355 041
44	49	2 094 384·69	31 472 117·98	94	99	13·284 074	34·310 645
45	50	2 017 561·97	29 377 733·29	95	100	9·061 652	21·026 571
46	51	1 946 490·10	27 360 171·32	96	101	5·957 791	11·964 919
47	52	1 875 053·78	25 413 681·22	97	102	3·626 626	6·007 128
48	53	1 803 411·85	23 538 627·44	98	103	1·860 174	2·380 502
49	54	1 731 684·82	21 735 215·59	99	104	0·520 328	0·520 328

SERIES V.—Assurance on (y) against (x).

Age of			Log. $M_{x,y}^{-1}$ — Log. $D_{x,y}^{-1}$ =	Age of			Log. $M_{x,y}^{-1}$ — Log. $D_{x,y}^{-1}$ =
(x)	(y)	Log. $M_{x,y}^{-1}$	Log. $A_{x,y}^{-1}$	(x)	(y)	Log. $M_{x,y}^{-1}$	Log. $A_{x,y}^{-1}$
0	5	7.007 5427	9.175 2254	50	55	6.220 3667	9.609 1570
1	6	6.958 6038	9.219 5030	51	56	6.201 5010	9.616 8500
2	7	6.927 5356	9.243 1373	52	57	6.181 8434	9.624 6124
3	8	6.906 5358	9.267 9622	53	58	6.160 5770	9.632 0073
4	9	6.891 5958	9.285 5281	54	59	6.136 2866	9.638 2664
5	10	6.880 5135	9.302 1498	55	60	6.108 2098	9.642 8836
6	11	6.871 0554	9.315 2839	56	61	6.075 1358	9.645 2944
7	12	6.861 1699	9.325 7016	57	62	6.039 9230	9.647 0760
8	13	6.851 1397	9.334 5224	58	63	6.003 2383	9.648 9579
9	14	6.840 9399	9.342 2829	59	64	5.965 9260	9.652 0625
10	15	6.830 2439	9.349 0431	60	65	5.927 4340	9.656 4891
11	16	6.818 4259	9.354 7132	61	66	5.888 1400	9.663 0462
12	17	6.805 7755	9.359 9216	62	67	5.848 1156	9.670 5472
13	18	6.792 8896	9.365 0631	63	68	5.807 0141	9.678 5593
14	19	6.780 0690	9.370 3705	64	69	5.764 7523	9.686 7289
15	20	6.767 3183	9.375 9189	65	70	5.720 8570	9.695 1652
16	21	6.754 6460	9.381 8579	66	71	5.675 5404	9.703 9357
17	22	6.742 3584	9.388 3576	67	72	5.624 6517	9.711 0871
18	23	6.730 1712	9.395 0691	68	73	5.566 2656	9.715 9008
19	24	6.718 0883	9.401 9275	69	74	5.499 5017	9.717 9550
20	25	6.706 1120	9.408 9357	70	75	5.421 9780	9.716 1767
21	26	6.693 9584	9.415 8848	71	76	5.339 1305	9.712 7979
22	27	6.681 9105	9.422 9129	72	77	5.248 6985	9.708 7370
23	28	6.669 4062	9.429 6808	73	78	5.153 4040	9.706 2825
24	29	6.655 6000	9.435 5759	74	79	5.056 6787	9.707 7538
25	30	6.640 1838	9.440 3778	75	80	4.951 4607	9.711 1449
26	31	6.624 5077	9.445 1370	76	81	4.844 2001	9.716 6911
27	32	6.608 8451	9.449 9787	77	82	4.726 9543	9.721 8631
28	33	6.593 4791	9.455 2585	78	83	4.604 7656	9.727 7203
29	34	6.578 4300	9.461 2283	79	84	4.474 4142	9.731 2736
30	35	6.563 4450	9.467 8018	80	85	4.338 1637	9.737 6888
31	36	6.548 5372	9.474 6189	81	86	4.187 9675	9.740 3911
32	37	6.533 4373	9.481 4185	82	87	4.021 3491	9.742 3718
33	38	6.518 1412	9.488 1216	83	88	3.832 1299	9.737 6436
34	39	6.502 6427	9.494 7245	84	89	3.637 6146	9.734 8072
35	40	6.486 3940	9.500 9341	85	90	3.447 4017	9.737 9175
36	41	6.468 8294	9.506 3694	86	91	3.209 3204	9.727 4663
37	42	6.450 4412	9.511 4496	87	92	2.934 1734	9.704 6589
38	43	6.431 4690	9.516 3418	88	93	2.651 3432	9.683 1372
39	44	6.412 4374	9.521 4040	89	94	2.379 9237	9.662 6981
40	45	6.393 3539	9.526 8213	90	95	2.103 4881	9.629 4287
41	46	6.374 5090	9.532 9780	91	96	1.833 8859	9.619 1562
42	47	6.355 9323	9.539 7454	92	97	1.586 0785	9.636 7694
43	48	6.337 9233	9.547 2497	93	98	1.343 2982	9.658 6383
44	49	6.321 0564	9.555 6940	94	99	1.123 3313	9.686 5777
45	50	6.304 8269	9.564 7599	95	100	0.957 2074	9.745 3799
46	51	6.289 2522	9.574 3688	96	101	0.775 0853	9.800 6331
47	52	6.273 0138	9.583 7023	97	102	0.559 5028	9.850 4711
48	53	6.256 0949	9.592 6620	98	103	0.269 5535	9.904 3522
49	54	6.238 4689	9.601 0379	99	104	9.716 2775	9.945 7700

SERIES V.—Assurance on (y) against (x).

Age of		Present value of an Assurance of £1 on the Life of (y), the oldest, against (x), the youngest.	Age of		Present value of an Assurance of £1 on the Life of (y), the oldest, against (x), the youngest.
(x)	(y)	$A_{x:y}^{-1}$	(x)	(y)	$A_{x:y}^{-1}$
0	5	·14 970 12	50	55	·40 659 03
1	6	·16 576 89	51	56	·41 385 67
2	7	·17 504 00	52	57	·42 132 03
3	8	·18 533 70	53	58	·42 855 57
4	9	·19 298 70	54	59	·43 477 68
5	10	·20 051 64	55	60	·43 942 38
6	11	·20 667 31	56	61	·44 186 99
7	12	·21 169 06	57	62	·44 368 63
8	13	·21 603 42	58	63	·44 561 31
9	14	·21 992 92	59	64	·44 881 00
10	15	·22 337 94	60	65	·45 340 79
11	16	·22 631 49	61	66	·46 030 55
12	17	·22 904 54	62	67	·46 832 49
13	18	·23 177 32	63	68	·47 704 50
14	19	·23 462 30	64	69	·48 610 37
15	20	·23 763 97	65	70	·49 563 87
16	21	·24 091 17	66	71	·50 574 98
17	22	·24 454 43	67	72	·51 414 68
18	23	·24 835 28	68	73	·51 987 72
19	24	·25 230 59	69	74	·52 234 20
20	25	·25 641 04	70	75	·52 020 76
21	26	·26 054 62	71	76	·51 617 61
22	27	·26 479 69	72	77	·51 137 21
23	28	·26 895 57	73	78	·50 849 01
24	29	·27 263 14	74	79	·51 021 56
25	30	·27 566 26	75	80	·51 421 51
26	31	·27 870 00	76	81	·52 082 41
27	32	·28 182 45	77	82	·52 706 36
28	33	·28 527 16	78	83	·53 422 02
29	34	·28 922 00	79	84	·53 860 90
30	35	·29 363 09	80	85	·54 662 41
31	36	·29 827 64	81	86	·55 003 60
32	37	·30 298 31	82	87	·55 255 03
33	38	·30 769 58	83	88	·54 656 73
34	39	·31 240 97	84	89	·54 300 93
35	40	·31 690 87	85	90	·54 691 20
36	41	·32 089 97	86	91	·53 390 79
37	42	·32 467 55	87	92	·50 659 27
38	43	·32 835 36	88	93	·48 210 01
39	44	·33 220 34	89	94	·45 993 67
40	45	·33 637 31	90	95	·42 601 87
41	46	·34 117 56	91	96	·41 606 02
42	47	·34 653 36	92	97	·43 328 08
43	48	·35 257 35	93	98	·45 565 72
44	49	·35 949 59	94	99	·48 593 44
45	50	·36 707 93	95	100	·55 639 08
46	51	·37 529 15	96	101	·63 187 79
47	52	·38 344 44	97	102	·70 871 42
48	53	·39 143 71	98	103	·80 232 86
49	54	·39 905 97	99	104	·88 261 24

TABLE A, for finding the value of Joint Annuities on (x) and (y).

Age of		$l_{x,y} \times v^x =$	Sum of $D_{x,y} =$	Age of		$l_{x,y} \times v^x =$	Sum of $D_{x,y} =$
(x)	(y)	$D_{x,y}$	$N_{x,y}$	(x)	(y)	$D_{x,y}$	$N_{x,y}$
0	5	67 970 000	1 045 627 530	50	55	4 085 166	44 624 730
1	6	54 840 430	977 657 530	51	56	3 842 828	40 539 564
2	7	48 350 200	922 817 100	52	57	3 607 705	36 696 736
3	8	43 508 450	874 466 900	53	58	3 377 301	33 089 031
4	9	40 371 080	830 958 450	54	59	3 147 895	29 711 730
5	10	37 875 960	790 587 370	55	60	2 919 619	26 563 835
6	11	35 956 010	752 711 410	56	61	2 690 552	23 644 216
7	12	34 313 760	716 755 400	57	62	2 470 853	20 953 664
8	13	32 856 200	682 441 640	58	63	2 260 895	18 482 811
9	14	31 525 140	649 585 440	59	64	2 059 982	16 221 916
10	15	30 283 130	618 060 300	60	65	1 866 143	14 161 934
11	16	29 087 920	587 777 170	61	66	1 679 167	12 295 791
12	17	27 916 050	558 689 250	62	67	1 505 110	10 616 624
13	18	26 780 980	530 773 200	63	68	1 344 172	9 111 514
14	19	25 686 120	503 992 220	64	69	1 196 805	7 767 342
15	20	24 626 310	478 306 100	65	70	1 060 942	6 570 537
16	21	23 593 270	453 679 790	66	71	936 709	5 509 595
17	22	22 594 400	430 086 520	67	72	819 529	4 572 886
18	23	21 632 270	407 492 120	68	73	708 540	3 753 356
19	24	20 709 080	385 859 850	69	74	604 709	3 044 816
20	25	19 823 310	365 150 770	70	75	507 927	2 440 106
21	26	18 970 270	345 327 460	71	76	422 992	1 932 179
22	27	18 155 050	326 357 190	72	77	346 706	1 509 187
23	28	17 367 020	308 202 140	73	78	279 976	1 162 481
24	29	16 596 790	290 835 120	74	79	223 318	882 504
25	30	15 841 850	274 238 330	75	80	173 906	659 186
26	31	15 113 700	258 396 480	76	81	134 124	485 279
27	32	14 416 720	243 282 780	77	82	101 179	351 154
28	33	13 747 410	228 866 060	78	83	75 343	249 975
29	34	13 097 900	215 118 650	79	84	55 352	174 632
30	35	12 463 592	202 020 753	80	85	39 854	119 279
31	36	11 855 457	189 557 161	81	86	28 026	79 425
32	37	11 272 462	177 701 704	82	87	19 009	51 398
33	38	10 715 676	166 429 242	83	88	12 430	32 388
34	39	10 183 996	155 713 566	84	89	7 994	19 957
35	40	9 670 744	145 529 570	85	90	5 122	11 963
36	41	9 171 914	135 858 826	86	91	3 032	6 840
37	42	8 689 436	126 686 912	87	92	1 696	3 807
38	43	8 224 836	117 997 476	88	93	929	2 111
39	44	7 780 964	109 772 640	89	94	521	1 181
40	45	7 354 152	101 991 676	90	95	297	660
41	46	6 942 742	94 637 524	91	96	163	362
42	47	6 549 179	87 694 782	92	97	88	198
43	48	6 175 521	81 145 603	93	98	48	109
44	49	5 825 891	74 970 082	94	99	27	61
45	50	5 496 256	69 144 191	95	100	16	33
46	51	5 186 607	63 647 935	96	101	9	17
47	52	4 890 030	58 461 328	97	102	5	8
48	53	4 607 155	53 571 298	98	103	2	2
49	54	4 339 413	48 964 143	99	104	0	0

NOTE.— $N_{x,y}$ is tabulated in the form which gives $\frac{N_{x,y}}{D_{x,y}} = 1 + a_{x,y} = \text{Annuity per advance.}$